Weak dynamical localization in periodically kicked cold atomic gases

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Quantum kicked rotor was recently realized in experiments with cold atomic gases and standing optical waves. As predicted, it exhibits dynamical localization in the momentum space. Here we consider the weak localization regime concentrating on the Ehrenfest time scale. The later accounts for the spread-time of a minimal wavepacket and is proportional to the logarithm of the Planck constant. We show that the onset of the dynamical localization is essentially delayed by four Ehrenfest times and give quantitative predictions suitable for an experimental verification.

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Unprecedented degree of control reached in experiments with ultra-cold atomic gases [1] allows to investigate various fundamental quantum phenomena. A realization of quantum kicked rotor (QKR) is one such possibility that recently attracted a lot of attention [2, 3, 4, 5]. To this end cold atoms are placed in a spatially periodic potential $V_0 \cos(2k_L x)$ created by two counter-propagated optical beams. The potential is switched on periodically for a short time $\tau_p \ll T$, giving a kick to the atoms; here T is a period of such kicks. The evolution of the atomic momenta distribution may be monitored after a certain number of kicks. If the gas is sufficiently dilute [6], one may model it with the single–particle Hamiltonian, that upon the proper rescaling takes the form [7] of the QKR:

$$\hat{H} = \frac{1}{2}\,\hat{l}^{\,2} + K\cos\theta \sum_n \delta\left(t-n\right) . \qquad (1)$$

Here $\theta \equiv 2k_L x$ and time is measured in units of the kick period, T. The momentum operator is defined as $\hat{l} = i\bar{k}\,\partial_\theta$, where the dimensionless Planck constant is given by $\bar{k} = 8\hbar T k_L^2/(2m)$. Finally, the classical stochastic parameter is $K = \bar{k} V_0 \tau_p/\hbar$.

The classical kicked rotor is known to have the rich and complicated behavior [8]. In particular, for sufficiently large K ($\gtrsim 5$), it exhibits the chaotic diffusion in the space of angular momentum [8]. The latter is associated with the diffusive expansion of an initially sharp momenta distribution: $\delta\langle l^2(t)\rangle \equiv \langle (l(t)-l(0))^2\rangle = 2D_{cl}t$ (dashed line on Fig. 1). For sufficiently large K, the classical diffusion constant may be approximated by $K^2/4$ [8]. The higher order correction is an oscillatory function of the stochastic parameter, i.e., $D_{cl}(K) \approx \frac{1}{4}K^2(1 - 3J_2(K) + 2J_2^2(K))$ [9, 10]. It was realized a while ago [11, 12] that quantum interference destroys the diffusion in the long time limit and leads to localization: $\delta\langle l^2(t)\rangle \to\sim \xi^2$ at $t \gtrsim t_L \equiv D_{cl}/\bar{k}^2 = \xi^2/D_{cl}$, where the localization length is given by $\xi = D_{cl}/\bar{k}$. For a large localization length $\xi \gg \bar{k}$, there is a long crossover regime, $1 < t < t_L$, between the classical diffusion and quantum localization.

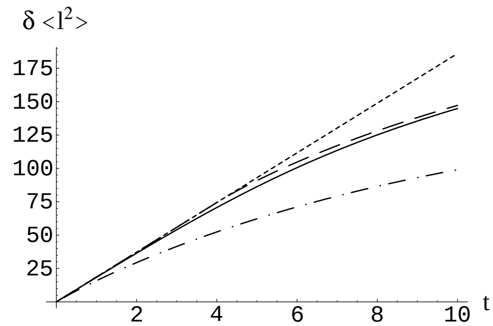

FIG. 1: The momentum dispersion for $K = 6.1$ and $\bar{k} = 0.6$ – full line; the classical limit ($\bar{k} \to 0$) – dashed line, standard weak localization ($t_E = 0$) – dashed-dotted line; the limit $\lambda_2 \to 0$, Eq. (3) – long-dashed line.

It was suggested in Ref. [13], that the QKR may be mapped onto the one–dimensional Anderson localization with the long range disorder. The universal long–time behavior of the latter is described by the non–linear sigma–model [14], resulting in the standard weak–localization correction [15, 16]: $\delta\langle l^2(t)\rangle = 2D_{cl}t\,(1 - 0.75\sqrt{t/t_L}\,)$ (dashed–dotted line on Fig. 1). Notice, that the correction is linear in $\bar{k}$ and non-analytic in time. In an apparent contradiction with this fact, explicit studies [17, 18] of the first few kicks show only renormalization of the diffusion constant starting from terms $\sim \bar{k}^2$. The aim of this paper is to develop a quantitative description of the classical to quantum crossover for the QKR that, in particular, accounts for these conflicting observations.

It is known in various contexts [19] that such crossover involves an additional time scale, t_E, called an Ehrenfest (or breaking) time. For a generic quantum mapping, it was first shown by Berman and Zaslavski [20, 21], that quantum corrections become comparable to the classical limit at the time t_E. This is the time needed for a minimal quantum wavepacket, $\delta\theta_0\delta l_0 \simeq \bar{k}$, to spread uniformly over the angular direction. Due to the chaotic mo-

tion, trajectories diverge as $\delta\theta(t) = \delta\theta_0 e^{\lambda t}$, where λ is the classical Lyapunov exponent. For $K \gg 1$, $\lambda = \ln(K/2)$ [8]. Estimating $\delta l_0 \approx K\delta\theta_0$ – a typical momenta dispersion after one kick, one finds for the Ehrenfest time:

$$t_E = \frac{1}{\lambda} \ln \sqrt{\frac{K}{\hbar}} \,. \tag{2}$$

It is widely believed [12, 20, 21, 22] that this intermediate, $1 < t_E < t_L$, time scale is indeed relevant for the quantum evolution of classically chaotic systems. This observation was put on the quantitative basis in Ref. [23] in the context of localization caused by classical scatterers. In this paper we adopt methods of Ref. [23] to the essentially different problem of the QKR.

In the leading order in $\hbar$ we found for the momentum dispersion:

$$\delta\langle l^2(t)\rangle = 2D_{cl}t - \frac{8\hbar\sqrt{D_{cl}}}{3\sqrt{\pi}}\,\theta(t-4t_E)\,(t-4t_E)^{3/2}\,, \tag{3}$$

where $\theta(t)$ is the step function (long-dashed line on Fig. 1). At relatively large time, $t_E \ll t < t_L$ our result approaches the standard weak–localization, mentioned above. However, corrections of the order of $\hbar$ are absent for $t \le 4t_E$. The delay is caused by the interference nature of the localization. Indeed, the first correction originates from the interference of two closed–loop counter–propagating trajectories, see Fig. 2. It takes time about t_E for classical trajectories passing through (almost) the same value of the momentum to diverge and take counter–propagating roots. As a result, the interference effects are practically absent at smaller times and show up only after $4t_E$.

One may show that the time interval $0 \le t \le 4t_E$ is protected from higher order weak localization corrections as well. For example, in the second order weak localization correction there are two diagrams [24] proportional to $\hbar^2\,\theta(t-mt_E)(t-mt_E)^2$ with $m = 6$ and $m = 8$ correspondingly. This fact agrees with the perturbative studies of the QKR dynamics after a few kicks [2, 11, 12], where no localization effects were seen. (Though the classical diffusion coefficient, D_{cl}, is renormalized as an analytic function of $\hbar^2$.) It is important to mention, however, that in reality there is no non–analyticity at the point $t = 4t_E$ as may seem from Eq. (3). The small localization corrections, non–analytic in $\hbar$, *do* exist for $t \lesssim 4t_E$. They are associated with the fluctuations of the Ehrenfest time. Since in the quantum mechanics the minimal separation between the trajectories is about $\delta\theta_0 \sim \sqrt{\hbar/K}$, it takes a finite time ($\sim t_E$) for them to diverge. This time may fluctuate depending on initial conditions. The fluctuations are characterized by the time scale $\delta t_E = \lambda_2 t_E/\lambda^2$, where (cf. Eq. (2)) $\lambda_2 = \langle\lambda^2\rangle - \langle\lambda\rangle^2 \approx 0.82$ for sufficiently large K and the angular brackets denote averaging over the initial angle. The interference between rare trajectories, diverging

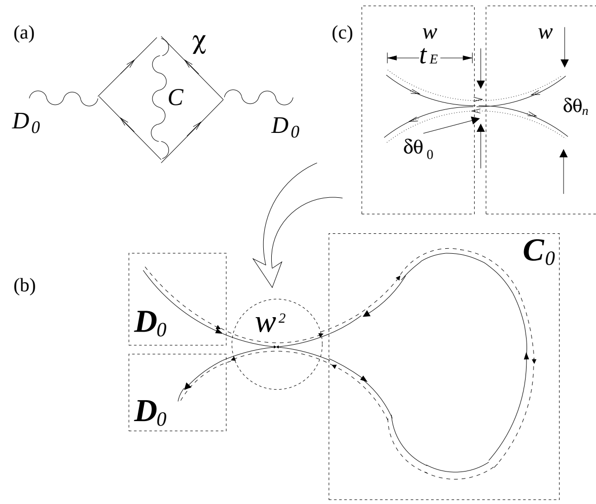

FIG. 2: The first quantum correction to the density–density correlator: (a) one–loop weak localization diagram; (b) its image in the momentum space; (c) semiclassical Hikami box.

faster than the typical ones, leads to quantum corrections at $t \lesssim 4t_E$ of the form:

$$\delta\langle l^2(t)\rangle = 2D_{cl}t - \frac{\Gamma\left(\frac{5}{4}\right)\hbar}{3\pi/256}\sqrt{D_{cl}}\,(\delta t_E)^{3/2} f\left(\frac{4t_E - t}{\sqrt{\delta t_E}}\right), \tag{4}$$

where $f(0) = 1$ and $f(x) = 6\sqrt{2\pi}/\Gamma\left(\frac{5}{4}\right) x^{-5/2} e^{-x^2/16}$ for $x \gg 1$. Localization correction including effect of the Ehrenfest time fluctuations is depicted on Fig. 1 by a full line. It is rather close to the prediction of Eq. (3) (long-dashed line), however the singularity at $t = 4t_E$ is rounded.

Having outlined our main results, we turn now to some details of the calculations. One starts from introducing the exact one period evolution operator as: $\hat{U} = \exp\left\{i(K/\hbar)\cos\hat{\theta}\right\}\exp\left\{i\hat{l}^2/(2\hbar)\right\}$. All physical quantities may be expressed in terms of the matrix elements of $\hat{U}^n$, where n stays for the number of kicks (time). We shall be particularly interested in the four–point density–density correlator, defined as:

$$\mathcal{D}(l_+, l_-; l'_+, l'_-; \omega_+, \omega_-) \tag{5}$$
$$\equiv \sum_{n,n'=0}^{\infty} \langle l_+|e^{i\omega_+ n} e^{\frac{i\hat{l}^2}{2\hbar}} \hat{U}^n |l'_+\rangle \overline{\langle l'_-|e^{i\omega_- n'} e^{\frac{i\hat{l}^2}{2\hbar}} \hat{U}^{n'} |l_-\rangle}\,,$$

where $|l_\pm\rangle$ denote momentum eigenstates. First we note that averaging over $(\omega_+ + \omega_-)/2$ leads to $n = n'$ [14]. Then performing the standard Wigner transform, one passes to the variables $l = (l_+ + l_-)/2$ and θ – the Fourier transform of $l_+ - l_-$ (and similarly for the prime variables). Since the most unstable direction of the underlying classical dynamics is along the θ direction [8], any initially smooth distribution quickly relaxes in this direction. Thus, averaging over θ and θ' may be performed.

The resulting correlator depends only on the relative momentum $l-l'$ and frequency $\omega \equiv \omega_+ - \omega_-$. Introducing finally angle φ as the Fourier image of $l-l'$, one ends up with $\mathcal{D} = \mathcal{D}(\varphi;\omega)$. Its classical limit, $\mathcal{D}_0(\varphi;\omega)$, may be found e.g. by using the diagrammatic technique of Ref. 15, where in the large K limit it corresponds to the family of ladder diagrams. Alternatively, one may show from Eq. (5) that $\mathcal{D}_0$ satisfies classical Liouville equation. Upon the proper regularization [9, 10, 25], that may be viewed as a coarse graining in the angular direction, one arrives to the classical diffusion propagator:

$$\mathcal{D}_0(\varphi;\omega) = \left(-i\omega + D_{cl}\varphi^2\right)^{-1} . \qquad (6)$$

This classical limit reflects the diffusion in the momentum space: $\delta\langle l^2(t)\rangle = 2D_{cl}t$, with the diffusion coefficient, $D_{cl}(K)$, studied extensively in the literature [8, 9, 10].

The first quantum correction to Eq. (6) is given by the one–loop weak–localization diagram, Fig. 2a. It describes the interference of the two counter–propagating trajectories, passing through (almost) the same point in the momentum space, Fig. 2b. In the Wigner representation such correction takes the form:

$$\delta\mathcal{D}(l,\theta;l',\theta') = \int\!\!\int \frac{dl_0 d\theta_0}{2\pi}\,\frac{dl_1 d\theta_1}{2\pi}\Big\{\mathcal{C}(l_1,\theta_0;l_0,\theta_1)$$
$$\times\ \hat{\mathcal{X}}(l_0,\theta_0;l_1,\theta_1)\big[\mathcal{D}_0(l,\theta;l_0,\theta_0)\,\mathcal{D}_0(l_1,\theta_1;l',\theta')\big]\Big\}, \quad (7)$$

where ω argument is omitted to shorten notations. The operator $\hat{\mathcal{X}}$ stays for the Hikami box [26], which is given by

$$\hat{\mathcal{X}} = -\exp\left\{-\frac{K^2(\delta\theta_0)^4}{4\hbar^2} + 4i\frac{\delta l_0\delta\theta_0}{\hbar}\right\} D_{cl}\left(\nabla^2_{l_0} + \nabla^2_{l_1}\right) , \qquad (8)$$

where $\delta\theta_0 \equiv \theta_0+\theta_1$ and $\delta l_0 \equiv (l_0-l_1)/2$. It is clear from this expression that the quantum correction, Eq. (7), is non–zero as long as $\delta l_0\delta\theta_0 \lesssim \hbar$ and, therefore, it is proportional to $\hbar$. The semiclassical Cooperon $\mathcal{C}(l_0,\theta_0;l_1,\theta_1)$ gives the probability of return to (almost) the same momentum, $l_1 \approx l_0$, at (almost) the opposite angle, $\theta_1 \approx -\theta_0$. If these conditions were strict, such motion would be forbidden by the time–reversal symmetry. The quantum uncertainty makes it possible. It takes, however, a long time to magnify the initially small angular variation $\delta\theta_0 \simeq \sqrt{\hbar/K}$ (this estimate as well as $\delta l_0 \simeq \sqrt{\hbar K}$ follows directly from Eq. (8)) up to $\delta\theta_n \approx 1$, when the usual diffusion takes place.

To take this fact into account we divide the Cooperon trajectory onto two parts: the Ehrenfest region, where $\delta\theta_n \ll 1$ and the diffusive region with $\delta\theta_n \gtrsim 1$. We denote the corresponding propagators as $\mathcal{W}$ and $\mathcal{C}_0$ and write in the time representation $\mathcal{C}(t) = \int dt'\,\mathcal{W}(t')\,\mathcal{C}_0(t-2t')$ (cf. Fig. 2b), where, as we show below, $t' \approx t_E$. Notice that the diffusive part of the trajectory is shortened by $2t'$, leading to $\mathcal{C}(\omega) = \mathcal{W}(2\omega)\,\mathcal{C}_0(\omega)$. The diffusive Cooperon, $\mathcal{C}_0(l_0-l_1;\omega)$, has the same form as Eq. (6) and thus $\mathcal{C}_0(0;\omega) \sim \int d\varphi (D_{cl}\varphi^2 - i\omega)^{-1}$.

To evaluate propagator $\mathcal{W}(2\omega)$ in the Ehrenfest regime, we define $\mathcal{W}(z,n)$ as a probability to reach the deviation $\delta\theta_n \equiv e^z$ during n kicks, starting from an initially small variation, $\delta\theta_0 \simeq \sqrt{\hbar/K}$. According to the classical equations (the standard map) $\theta_n = \theta_{n-1} + l_n$ and $l_n = l_{n-1} + K\sin\theta_{n-1}$, the variation evolves as $\delta\theta_n = \delta\theta_{n-1}(1+K\cos\theta_{n-1}) + 2\delta l_{n-1}$. Since $\delta l_0 \simeq K\delta\theta_0$, in the leading order in $K \gg 1$ the evolution of $\delta\theta$ is given by $\delta\theta_n \approx \delta\theta_0\prod_{j=0}^{n-1} K\cos\theta_j$. Taking the logarithm of this expression, one obtains

$$\mathcal{W}(z;n) = \left\langle \delta\left(z - \ln|\delta\theta_0| - \sum_{j=0}^{n-1}\ln|K\cos\theta_j|\right)\right\rangle, \quad (9)$$

where the averaging is performed over the initial distribution of $\delta\theta_0$ [with the typical scale $\delta\theta_0 \sim \sqrt{\hbar/K}$, cf. Eq. (8)] as well as over dynamics of the fast variable, that is $\delta l_n/\delta\theta_n$. For $K \gg 1$, one may treat $\cos\theta_j$ after successive kicks as independent random variables and employ the central limiting theorem to perform the averaging in Eq. (9). As a result,

$$\mathcal{W}(z;n) \approx \exp\left\{-\frac{(z - \ln\sqrt{\hbar/K} - n\lambda)^2}{2n\lambda_2}\right\} , \qquad (10)$$

where the Lyapunov exponent λ [8] and its dispersion λ_2 are defined as:

$$\begin{aligned}\lambda &\equiv \langle \ln|K\cos\theta|\rangle = \ln(K/2)\,; \qquad (11)\\ \lambda_2 &\equiv \langle \ln^2|K\cos\theta|\rangle - \lambda^2 = \zeta(3) - \ln^2 2 \approx 0.82\,;\end{aligned}$$

the angular brackets imply integration over θ. The Ehrenfest evolution crosses over to the usual diffusion at $\delta\theta_n \approx 1$, meaning $z \approx 0$. Performing finally the Fourier transform as $\mathcal{W}(\omega) \equiv \sum_n e^{i\omega n}\mathcal{W}(0,n)$ and employing the definition of the Ehrenfest time, Eq. (2), and the fact that $\lambda_2 \ll \lambda$, one finds:

$$\mathcal{W}(2\omega) = \exp\left\{2i\omega t_E - \frac{2\omega^2\lambda_2 t_E}{\lambda^2}\right\} . \qquad (12)$$

Due to the time–reversal invariance there is an exact symmetry between divergence and convergence of the two classical trajectories involved in the weak–localization correction. This symmetry is illustrated on Fig. 2c. Therefore it takes an additional time $\sim t_E$ for the two distinct semiclassical diffusons to arrive to the point $l_0 \approx l_1$ and $\theta_0 \approx -\theta_1$, bringing, thus, another factor $\mathcal{W}(2\omega)$. In a slightly different language, one may define the Hikami box for a classically chaotic system [23, 27] as $2\hbar\mathcal{W}^2(2\omega)D_{cl}\nabla_l^2$, where one factor $\mathcal{W}(2\omega)$ comes from the two legs of the Cooperon, while another originates from the two diffusons. Finally, the quantum correction, Eq. (7), reduces to the renormalization of the diffusion coefficient in the classical propagator, Eq. (6), as

$D(\omega) = D_{cl} + \delta D(\omega)$ with

$$\delta D(\omega) = -\frac{\bar{k} D_{cl}}{\pi} \mathcal{W}^2(2\omega) \int \frac{d\varphi}{-i\omega + D_{cl}\varphi^2} \,. \tag{13}$$

Equations (12) and (13) constitute the main analytical results of this work. They describe quantitatively dynamical weak–localization of the QKR with the account for the Ehrenfest time phenomena. One may finally express the time evolution of the momentum dispersion in terms of the frequency–dependent diffusion coefficient. The exact relation reads as:

$$\delta\langle l^2(t)\rangle = \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \, \frac{1 - e^{-i\omega t}}{\omega^2} \, D(\omega) \,. \tag{14}$$

Neglecting fluctuations of the Ehrenfest time ($\lambda_2 \to 0$ in Eq. (12)) and performing frequency and angle integrations in Eqs. (14), (13), one obtains Eq. (3) for the momenta dispersion. Notice that in this approximation the evolution is purely classical at $t \le 4t_E$. To account for the quantum corrections at $t \lesssim 4t_E$ one needs to keep the λ_2 term in Eq. (12). The straightforward integration leads to Eq. (4).

Our results, Eqs. (3), (4), are expected to be quantitatively accurate if separations between the relevant time scales: $1 < t_E < t_L$ are large enough. This is the case when the two dimensionless constants satisfy inequalities: $\bar{k} < 1 < K$. (In the experiments, we are aware of [2, 3], $\bar{k} \gtrsim 2$, and thus $t_E \lesssim 1$.) Another restriction has to do with the dephasing time, τ_ϕ [28]. The later may originate from non–perfect periodicity of the kicks (noise) [29], spontaneous emission [3], as well as from many–body collisions between the atoms [6]. Whatever the nature of the dephasing time, one needs to ensure $t_E < \tau_\phi$ to observe the weak localization.

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